Separable Eqns Example

1. Solve $y' = \frac{x^2}{y}$

Soln:

$$\frac{dy}{dx} = \frac{x^2}{y}$$

$$y\,dy = x^2\,dx$$

$$\int y\,dy = \int x^2\,dx$$

$$\frac{y^2}{2} + C_1 = \frac{x^3}{3} + C_2$$

$$\frac{y^2}{2} - \frac{x^3}{3} = C$$

2. Solve $y' + y^2 \sin x = 0$

Soln:

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\frac{-1}{y^2}\,dy = \sin x\,dx$$

$$\int -y^{-2}\,dy = \int \sin x\,dx$$

$$y^{-1} + C_1 = -\cos x + C_2$$
$$y^{-1} + \cos x = C$$

3. Solve $y' = \cos^2(x) \cdot \cos^2(2y)$

Soln:

$$\frac{dy}{dx} = \cos^2(x) \cdot \cos^2(2y)$$

$$\frac{1}{\cos^2(2y)}\, dy = \cos^2(x)\, dx$$

$$\int \frac{1}{\cos^2(2y)}\, dy = \int \cos^2(x)\, dx$$

$$\frac{\tan(2y)}{2} + C_1 = \frac{\cos(x)\sin(x) + x}{2} + C_2$$

$$\frac{\tan(2y) - \cos(x)\sin(x) - x}{2} = C$$

4. Solve $y' = \dfrac{x - e^{-x}}{y + e^{y}}$

Soln:

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}$$

$$y + e^{y}\, dy = x - e^{-x}\, dx$$

$$\int y + e^{y}\, dy = \int x - e^{-x}\, dx$$

$$\frac{y^2}{2} + e^{y} + C_1 = \frac{x^2}{2} + e^{-x} + C_2$$

5. Solve $xy' = (1-y^2)^{1/2}$

Soln:

$$x \frac{dy}{dx} = \sqrt{1-y^2}$$

$$\frac{1}{\sqrt{1-y^2}}\, dy = \frac{1}{x}\, dx$$

$$\int \frac{1}{\sqrt{1-y^2}}\, dy = \int \frac{1}{x}\, dx$$

$$\arcsin(\quad y) - \ln|x| = C$$